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CEBAF Q & E Analysis
A Software Module for SRF Cavity Testing in the CEBAF VTA

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The CEBAF Q & E Analysis program is used to automate the rf testing of CEBAF superconducting rf (SRF) cavities with the Production Testing RF System in the Vertical Test Area. It was developed between January 1990 and May 1991 for use in production testing of cavity pairs for the CEBAF recirculating linear accelerator. It is composed of modules which provide rf control, data acquisition, analysis, display, and logging. The program language is “G”, which is the graphical data flow programming language developed by National Instruments, Inc. for LabVIEW™. In execution, compiled G code is equivalent to compiled C code. The program runs on Macintosh II or Quadra computer families, and requires three Nubus interface cards proprietary to National Instruments. These cards provide GPIB communication, digital and analog I/O, and DMA operations. Inter-application communications over the AppleTalk LAN enables programmatic data transfer to and from the cryogenic control computers. With the introduction of LabVIEW 3, the code is portable to the Windows or Sun workstation versions of LabVIEW.

Introduction to LabVIEW

Every module in LabVIEW has two windows, the Front Panel and the Block Diagram. The Front Panel takes its name from the basic metaphor used by LabVIEW, the Laboratory Virtual Instrument Engineering Workbench. Via the software, data acquisition, control, and analysis functions are integrated and customized to create a virtual bench top test instrument, i.e. a Virtual Instrument (VI). The Front Panel window is the virtual front panel of this instrument. It may contain buttons, knobs, switches, meters, graphical displays, indicators and the like. Objects on the Front Panel are either controls or indicators, which are equivalent to the input and output parameters respectively of the software module.

The Block Diagram of a Virtual Instrument *is* the program. In the Block Diagram, inputs, which appear as controls on the Front Panel, are data sources, and outputs, which appear as indicators on the Front Panel, are data sinks. Data flows from sources to sinks via assorted function modules and structures. These function modules may be an assembly of some library functions ranging from simple arithmetic to complex statistical analysis, DSP and hardware I/O operations. In addition, previously constructed virtual instruments may themselves be used as functions, making the architecture truly hierarchical. For-loop, While-loop, Case, and Sequence structures together with shift registers, local variables and global variables make the programming environment analogous to other high-level languages.

In data-flow programming, data flows via “wires” which the programmer draws between block diagram elements. The data type (e.g. integer or array) is represented by the line style. Each program element, whether structure or function, begins execution when data has “arrived” at all of its inputs and completes when all of its outputs have received data. Upon completion, data is available at output terminals to flow on to other functions.

Basic Cavity Measurements

CEBAF Q&E Analysis is a LabVIEW VI used to measure the characteristic unloaded Q (Q_o) and accelerating gradient (E_{acc}) performance of CEBAF SRF cavities. An rf signal derived from a VCO, conditioned by a vector modulator, and boosted by an amplifier is applied to the input coupler of a cavity cooled to liquid helium temperatures. The frequency, f, of the VCO (approximately 1497 MHz) is phase locked on the cavity transmitted signal. Measurement of the relative amplitudes of the incident power (P_i) and reflected power (P_r) at the input determines β_i, the degree of coupling of the input transmission line to the cavity resonance. The loaded quality factor of the resonance (Q_L) is determined by measuring the cavity decay time τ , which typically is in the range of 20 - 600 ms.

$$Q_L = \frac{f}{BW} = 2\pi f\tau \quad > 10^8$$

Q_o is then obtained by $Q_o \equiv \dfrac{2\pi fU}{P_d} = \left(1+\beta_i+\beta_t\right)Q_L \quad \left(1+\beta_i\right)2\pi f\tau \quad \left(\beta_t << 1\right)$,

where U is the cavity stored energy, P_d is the power dissipated in the cavity and β_t is the coupling of the transmission port to the cavity. Another useful parameter is Q_{et}, the external Q of the transmission port.

$$Q_{et} \equiv \frac{2\pi fU}{P_t} = \frac{Q_o}{\beta_t}$$

where P_t is the power coupled out this transmission line. Balancing the power accounting leads to the expression

$$U = \left(1+\beta_i\right)\tau\left(P_i - P_r - P_t\right) = \frac{Q_{et}\,P_t}{2\pi f}$$

The effective accelerating gradient of a cavity is proportional to the square root of the stored energy in the fundamental mode.

$$E_{acc} = C_o\sqrt{U} = C_o\sqrt{\frac{Q_{et}\,P_t}{2\pi f}}$$

Similarly, we may write

$$Q_o = \frac{Q_{et}\,P_t}{P_i - P_r - P_t}$$

These parameters are critical in the operation of CEBAF, because the heat load to the 2.0 K refrigeration system is inversely proportional to Q_o for a given accelerating voltage, and the attainable values of E_{acc} limit the deliverable beam energy of the accelerator. C_o, which for CEBAF cavities is $4.26\ MV/J^{1/2}$ is calculated using design codes such as URMEL or Superfish and verified by actual energy gain of an accelerated electron beam. The external Q of the transmission probe is a function only of the geometry of the pick-up probe and is a calibration constant needed by the CEBAF acceleration rf control system to determine the effective operating state of the cavity. Once Q_{et} is measured with decay methods, the relations above are used to make cw measurements of Q_o and E_{acc} These measurements and the associated data analysis are accomplished with CEBAF Q & E Analysis.

Operation of CEBAF Q & E Analysis

This section describes the overall flow of activity within CEBAF Q & E Analysis in order to orient the reader on first approach to the program. At version 40, CEBAF Q & E Analysis contains 105 sub-VIs. Detailed flow-diagrams of the program are provided by a printout of the program modules themselves. User procedures are provided elsewhere and require no knowledge of LabVIEW and relatively little experience with SRF cavities. See Figure 1, "Front Panel of CEBAF Q & E Analysis" on page 4.

Overview

Upon execution of the Virtual Instrument, the cable calibration global array is checked for valid data. If it is empty, global variables and I/O boards are initialized. The program then enters the outer While-loop. See Figure 1, "Front Panel of CEBAF Q & E Analysis" on page 4. Here, the VI Production RF Select reads the digital input lines that signal to which of six cavity testing positions the rf system is connected. The operator has entered the name of each cavity to be tested in a string array control on the Front Panel. The element of this array corresponding to the selected testing position is passed to the outer sequence structure which has two frames. The first frame contains the VI Retr Logged Q&E&T Data. This VI receives the cavity name as input, retrieves from the log file of that name all past data sets, and calculates the resultant values for Q_o and E_{acc}. The sub-VI Qo vs. Eacc, a global graph variable, is initialized with this data. This global variable maintains the content of the Q vs. E Graph on the Front Panel. The Q_o and E_{acc} arrays are passed to Frame 1 where they initialize shift registers of the middle While-loop. The program idles in this While-loop monitoring for operator action. A Front Panel indicator light toggles off and on with each pass through this While Loop.

The Front Panel presents the operator with, among other things, 12 “push-button” switches. The corresponding boolean variables in the Block Diagram are wired to the case select inputs of True/False Case structures. The normally selected False cases are empty. When the operator changes one of the boolean variables to True, the True case executes, which typically contains a VI which directly accomplishes some procedure. We will return to discuss these VI's below. The operator is also presented with two analog controls on the Front Panel: Attenuation and Phase across the vector modulator. On each pass through the middle While-loop, the current input value for these parameters is passed to the Vector Modulator and VCO Tuning VI's and they execute. The phase and frequency controls may be disabled to allow optimization routines to set the desired values. Each cycle through the middle While-loop also includes a check to see if the cavity selection has been changed. If it has, the loop completes, which also completes the outer sequence structure, and the outer While-loop beings another cycle as its Do-while control is wired True with a constant.

The operator may initiate acquisition of a set of measurements by clicking on the Measure and Log button. Alternatively, the Auto Step Power Measure and Log button initiates a series of measurements with the attenuation of the vector modulator reduced 0.25 dB between successive acquisitions. A third nested While-Loop structure is employed to accomplish this. The number of measurements in the series is a Front Panel control. If for some reason the rf control system loses lock and the transmitted power falls to zero, the series is aborted by the Auto Stop Stepping VI.

The inner While-loop always executes at least once, even if no cavity performance measurements are to be made. It contains a three-frame sequence structure. Frame 0 provides an optional 2 minute dwell time for use when a series of measurements is to be spread out over an extended time, such as during the pumpdown of the dewar from 4.2K to 2.0K. In Frame 1, data about the cavity environment are acquired. This includes bath temperature and pressure, and the x-radiation level observed on a sensor above the cryostat but inside the shielding. If a value for Q_{et} has been entered in the control on the Front Panel, the incident, reflected and transmitted power meters are read and the values of Q_o and E_{acc} are calculated and displayed. Whenever the incident power level is greater than 10 mW, the Periodic RF Logging VI logs, at 30 second intervals, the cavity rf performance status, including the measured radiation level, to a separate log file associated with the cavity currently under

Figure 1Front Panel of CEBAF Q & E Analysis

Figure 2 One level of Block Diagram from CEBAF Q&E Analysis test. Frame 2 contains a True/False Case structure which is normally False. If the operator selects Clear Graph, the Q_o and E_{acc} arrays are reset to a single 0.0 element each and the Qo vs. Eacc global graph variable is emptied, clearing the Q vs. E Graph on the Front Panel. The True case contains all of the actual cavity measurements, data analysis and logging. This case is selected by the Measure and Log or Auto Step Power Measure and Log Front Panel boolean controls being set True.

This True case contains a three-frame Sequence structure. The great bulk of the activity takes place in Frame 0. For initial measurements of a high-Q SRF cavity, the value of Q_{et} is not yet known. It is determined from a cavity decay measurement which is accomplished by the VI Pt Decay Aqu & Analy 5. The operation of this VI is described later. It has several outputs which include cavity rf frequency (MHz), incident, reflected, and transmitted measured cw power levels (W), two graphs, one of the Ptrans decay and the other of Q_l vs. time during the decay, and the decay time constant, τ. (For historical reasons CEBAF Q&E Analysis and its sub-VIs use the parameter $t_{1/3} = \tau \ln 3$.) If Q_{et} has been previously measured and the value enter in the control on the Front Panel, then only the cw power levels and the rf frequency are measured. For all measurements, the operator must determine whether the input coupling coefficient is greater or less than one and set the Front Panel control accordingly.

All of these parameters, together with the appropriate cable calibration constants that are retrieved by the VI Current Cals, are bundled into a cluster and provided as input to the analysis VI Stndcalc-ex2u. This VI calculates the desired cavity parameters such as Q_0, E_{acc}, β's, Q_{ext}'s, and P_l. The propagation of errors associated with the derivation of each of these parameters is performed in the sub-VIs ERRORS CALC and ERRORS CALC CW2. The output of Stndcalc-ex2u is a cluster containing the parameter values and their estimated percentage uncertainty. This cluster is wired to an indicator on the Front Panel of CEBAF Q&E Analysis in order to provide immediate feedback to the operator. New elements with the derived values of Q_0 and E_{acc} are prepended to the respective arrays, and a new point is added to the Q vs. E graph on the Front Panel. The cavity surface resistance is calculated and a point is added to the R_s vs. T_c/T graph on the Front Panel. The analysis equations used in CEBAF Q&E Analysis are presented beginning on page 10.

In Frame 1 of the Sequence, a cluster is formed and the information for the data point is optionally sent immediately to a printer. Frame 2 constructs a cluster containing just the raw data and writes it to a log file via the sub-VI log CW Power data 3. (It is this log file that serves as the record of cavity performance. Subsequent analysis always reanalyzes the basic data.) This completes the inner While-loop.

Supporting Functions

In support of the cavity testing process, several sets of operations are available for the convenience of the operator. This section includes introductions to several of them.

Cable Calibration

The basic rf power measurements are made at power sensors located in the control room. These measurements must be referred to the respective traveling wave power at the cavity coupling ports. When the VSWR of line elements is kept low and cable attenuations are constant, simple calibration constants are all that are needed. The procedure for determining these constants is given in the use procedures manual. Five pairs of power measurements are required to determine the three constants. The software tools used in this process are outlined here.

By selecting the Auto Calibrate Cables button on the Front Panel, the operator executes the Auto Cable Cal VI. This opens a new window on the monitor which presents the operator with five measurements options and a control for the vector modulator attenuation. See Figure 3, "Front Panel of Auto Cable Cal" below. An indicator displays the currently selected cavity testing position. An array containing the latest measurements for all of the cavity positions is also displayed. This information is maintained in a global array VI.

When the rf hardware is appropriately arranged, the operator clicks one of the buttons which executes a measurement VI such as Trans Ins Loss Meas that in turn prompts the operator to enter the applied power as read on a remote power meter and then automatically reads the local transmitted power meter and enters the results in the array of measurements. When the measurements are complete, the operator selects the Stop button and the data is routed to the Calc calibs VI for reduction and posting to a global variable array. As an illustration of the LabVIEW programming style, the Block Diagram of Auto Calibrate Cables is presented in Figure 4.

Phase Optimization For The PLL

For one to obtain good measurements of cavity parameters, the rf frequency must be tuned to the center of the resonance. Since the bandwidth is typically less than 0.5 Hz, a phase-lock-loop is required. The frequency of the VCO is phase locked to the cavity transmitted signal. The phase is trimmed to the appropriate setting by the vector modulator. The best coupling to the resonance is demonstrated by maximizing the transmitted power. For operation near critical input coupling, however, it is more practicable to adjust the phase so as to minimize the reflected power—if there are no spurious line reflections, the criterion are indistinguishable. Since coupling factor determinations are based on reflection measurements, the overall uncertainties in cavity performance measurements are minimized by optimizing the phase with reference to the reflected power. This process is automated by the VI Phase opt, which is available to the operator via a Front Panel button.

Figure 3 Front Panel of Auto Cable Cal

Figure 4. Block Diagram of Auto Calibrate Cables

Phase opt steps the phase over a range of ± 10° from the starting value while monitoring the reflected power amplitude on a crystal detector. A quadratic fit is performed on the data and a minimum sought. The optimization continues until the minimum is clearly within the search range and the phase is set to that value. The process takes 3 to 6 seconds, with the speed being limited by the cavity time constant.

Automatic Optimization of PLL Gain

The optimum operation of the PLL is dependent on having high loop gain, G_{PLL}, while yet remaining stable. The maximum stable gain for the circuit constructed was $G_{PLL} = 8.7 \times 10^5$. The capture frequency range varies like $\sqrt{G_{PLL}}$ with respect to the loop gain, while the lock range is proportional to the loop gain. G_{PLL} is in turn proportional to the square-root of the amplitude of transmitted power applied to the PLL mixer, which basically functions as a synchronous phase detector. This transmitted power can, however, vary by over 50 dB during cw testing of an SRF cavity—signal acquisition requires use of an additional 40 dB at the low end. In order to maintain a fairly constant -28 dBm input to the mixer, step attenuators and a low noise amplifier may be switched in or out of the transmission line between the cavity and the mixer. A complication is that the actuation of one of these switching functions also introduces a discrete phase shift. The operator has available an automatic optimization routine, the VI Trim Pt and LNA, which is selected by clicking on the Front Panel button Set Mixer Level. Trim Pt and LNA is automatically called when Auto Step Power Measure and Log steps up the input power.

Trim Pt and LNA makes a series of switching changes to keep the PLL gain nearly constant. First the transmitted power meter is read, and if $P_t < 40$ μW the low noise amplifier is switched in. The sub-VI Set Pt Xtal Level is called to trim the step attenuators. This set of attenuators has a range of 22 dB, in 1 dB steps. Set Pt Xtal Level checks to see if the loop is in oscillation. If so, the LNA is switched out and/or attenuation is added. With the loop stable, the attenuation is adjusted to bring -25 dBm to the P_t crystal detector and -28 dBm to the mixer. After this is accomplished, Phase opt again trims the phase.

Transient Decay Measurements

The VI Pt Decay Aqu & Analy 5 performs the critical decay measurements used to characterize the high Q-values of the SRF cavities. The VI consists of a 15-frame Sequence structure. It is called when the cavity is operating cw and the transmission probe Q_{ext} is not yet calibrated. Pt Decay Aqu & Analy 5 begins by reading the current power levels and cavity frequency. The gain of the P_t crystal detector ADC is adjusted to provide the maximum number of useful bits. A triggered DMA acquisition is set up for this channel with 6000 points and 0.5 ms/sample. The trigger is set at 95% of the current cw level. Next, the PIN switch on the cavity drive line is opened by GPIB command to a pulse generator. The Pt signal is captured as the stored energy in the cavity rings down.

The array of data is truncated at the 20% level and several analyses made on the data. A least-squares fit to an exponential decay is performed which produces the decay parameter Tau 0.95–0.20. In addition, the sub-VI Multi Fit delta Flex does a piece-wise fit to each 10 ms of data, yielding an array of exponential decay slopes. If there is at least 70 ms of data, the first six elements of the array are fitted with a second-order polynomial and extrapolated to obtain the slope at 100% of the cw level. This procedure helps to highlight non-linear loading that sometimes occurs at high field levels—peak surface fields of 20–60 MV/m—due to electron field emission inside the cavity.

The acquired data is combined with the 95–20% fit and the extrapolated 100% fit in a graph passed to the Front Panel. In addition, the array from Multi Fit delta Flex is translated into loaded Q versus time and also presented in a plot on the Front Panel. The same information can be translated into Q_0 versus E_{acc} mapped out during the single decay.

Pulsed Operation

It is often very useful to study the response of an SRF cavity to pulsed operation. This is especially true when some irregular phenomena is encountered. Within CEBAF Q&E Analysis, this utility is provided to the operator through selection of the button Pr and Pt Scope. When this button is selected, the VI strip executes and opens its Front Panel on the monitor. This VI adjusts the ADC gains of the P_r and P_t crystal detectors and opens a window to permit the operator to select the desired repetition rate and duty cycle—defaults are 0.2 Hz and 50%. This is accomplished with a modified version of the VI HP 8116A FG_Pulse Train which was in the VI library distributed with LabVIEW. The VI strip then provides a real-time trace of the two analog signals P_r and P_t.

Concluding Comments

This note has attempted to introduce the operation of CEBAF Q&E Analysis. Specific technical details may be obtained best by viewing the program directly. The programming environment provided by LabVIEW proved well suited to the task. Indeed, we simply would not have had the resources to implement the degree of automation and user convenience features that we did were it not for this programming environment. Had these features not been available, though, production testing of CEBAF SRF cavities would have required four to six times the amount of labor actually used.

Additional return on the software investment continues. Many of the sub-VIs have been incorporated into cavity testing routines on Sparc stations and Macintoshes at DESY, and are thus providing indirect support to the TESLA Test Facility. Also, minimal effort has been required to create variant versions of the code for specific R&D applications at CEBAF, including the window arcing studies.

CEBAF Vertical Pair Testing RF Performance Characterization
Decay Measurement Formulas

Starting Parameters:

P_{im}, P_{rm}, P_{tm} − Actual Measured Power (Watts)
C_i, C_r, C_t − Cable Calibration Factors relating measured Power to Associated Power at the Cavity
C_β − Over / Under coupling factor $= \pm 1$
τ − Decay fit Parameter at Field of CW Measurements (Typically 100-700 ms)
$F(\mathrm{Hz})$ − Cavity Resonant Frequency (1497.0 MHz)

Derivation of Performance Parameters:

$$P_{incident} = P_{im} C_i$$
$$P_{reflected} = P_{rm} C_r$$
$$P_{transmitted} = P_{tm} C_t$$
$$P_{loss} = P_{incident} - P_{reflected} - P_{transmitted}$$
$$|\Gamma| = \sqrt{P_{reflected} / P_{incident}}$$
$$\Gamma = C_\beta |\Gamma|$$
$$\beta^* = \frac{1-\Gamma}{1+\Gamma}$$
$$Q_l = 2\pi F \tau$$
$$Q^* = \frac{1+\beta^*}{Q_l}$$
$$Q_1 = Q^* / \beta^*$$
$$\beta_2 = P_{transmitted} / P_{loss}$$
$$\beta_1 = \beta^*(1+\beta_2)$$
$$Q_0 = (1+\beta_1+\beta_2) Q_l$$
$$Q_2 = Q_0 / \beta_2$$
$$E_{acc}(\mathrm{MV/m}) = 43.79 \times 10^{-6} \sqrt{Q_2 P_{transmitted}}$$
$$U(Joules) = \frac{Q_2 P_{transmitted}}{2\pi F}$$

CW Formulas

Starting Parameters:

P_{im}, P_{rm}, P_{tm} − Actual Measured Power (Watts)
C_i, C_r, C_t − Cable Calibration Factors relating measured Power to Associated Power at the Cavity
C_β − Over / Under coupling factor $= \pm 1$
Q_2 − Transmission Probe External Q Determined from previous Decay measurement
$F(\mathrm{Hz})$ − Cavity Resonant Frequency (1497.0 MHz)

Derivation of Performance Parameters:

$$P_{incident} = P_{im} C_i$$
$$P_{reflected} = P_{rm} C_r$$
$$P_{transmitted} = P_{tm} C_t$$
$$P_{loss} = P_{incident} - P_{reflected} - P_{transmitted}$$
$$|\Gamma| = \sqrt{P_{reflected} / P_{incident}}$$
$$\Gamma = C_\beta |\Gamma|$$
$$\beta_1 = \frac{1-\Gamma}{1+\Gamma}$$
$$Q_0 = \frac{Q_2 P_{transmitted}}{P_{loss}}$$
$$Q_1 = Q_0 / \beta_1$$
$$\beta_2 = Q_0 / Q_2$$
$$Q_l = \frac{Q_0}{1+\beta_1+\beta_2}$$
$$E_{acc}(\mathrm{MV/m}) = 43.79 \times 10^{-6} \sqrt{Q_2 P_{transmitted}(W)}$$
$$U(Joules) = \frac{Q_2 P_{transmitted}}{2\pi F}$$

Derivation of Measurement Errors - Decay Measurement:

Starting Parameters:

$P_{\min} -$ Sensitivity Limit of Power Sensors Used

$\Delta P_{cal} -$ Absolute Error in Measured Power Level

$\Delta C -$ Uncertainty in Cable Calibrations

Parameter Uncertainties:

$$\Delta P_{im} = P_{im} \times \Delta P_{cal} + P_{\min}$$

$$\Delta P_{rm} = P_{rm} \times \Delta P_{cal} + P_{\min}$$

$$\Delta P_{tm} = P_{tm} \times \Delta P_{cal} + P_{\min}$$

$$\Delta P_{incident} = P_{incident}\sqrt{(\Delta C)^2 + \left(\frac{\Delta P_{im}}{P_{im}}\right)^2}$$

$$\Delta P_{reflected} = P_{reflected}\sqrt{(\Delta C)^2 + \left(\frac{\Delta P_{rm}}{P_{rm}}\right)^2}$$

$$\Delta P_{transmitted} = P_{transmitted}\sqrt{(\Delta C)^2 + \left(\frac{\Delta P_{tm}}{P_{tm}}\right)^2}$$

$$\Delta|\Gamma| = \frac{|\Gamma|}{2}\sqrt{\left(\frac{\Delta P_{reflected}}{P_{reflected}}\right)^2 + \left(\frac{\Delta P_{incident}}{P_{incident}}\right)^2}$$

$$\Delta\beta^* = \sqrt{\left(\frac{\Delta|\Gamma|}{1+|\Gamma|}\right)^2 + \left(\frac{\Delta|\Gamma|}{1-|\Gamma|}\right)^2}$$

$$\Delta Q_l = Q_l \times \Delta\tau$$

$$\Delta Q^* = Q^*\sqrt{\left(\frac{\Delta\beta^*}{1+\beta^*}\right)^2 + \left(\frac{\Delta Q_l}{Q_l}\right)^2}$$

$$\Delta Q_1 = Q_1\sqrt{\left(\frac{\Delta Q^*}{Q^*}\right)^2 + \left(\frac{\Delta\beta^*}{\beta^*}\right)^2}$$

$$\Delta Q_0 = Q_0\sqrt{\left(\frac{\Delta\beta_1{}^2 + \Delta\beta_2{}^2}{\left(1+\beta_1+\beta_2\right)}\right) + \left(\frac{\Delta Q_l}{Q_l}\right)^2}$$

$$\Delta Q_2 = Q_2\sqrt{\left(\frac{\Delta Q_0}{Q_0}\right)^2 + \left(\frac{\Delta\beta_2}{\beta_2}\right)^2}$$

$$\Delta E = \frac{E}{2}\sqrt{\left(\frac{\Delta Q_2}{Q_2}\right)^2 + \left(\frac{\Delta P_{transmitted}}{P_{transmitted}}\right)^2}$$

$$\Delta P_{loss} = \sqrt{(\Delta P_{incident})^2 + (\Delta P_{reflected})^2 + (\Delta P_{transmitted})^2}$$

$$\Delta \beta_2 = \beta_2 \sqrt{\left(\frac{\Delta P_{transmitted}}{P_{transmitted}}\right)^2 + \left(\frac{\Delta P_{loss}}{P_{loss}}\right)^2}$$

$$\Delta \beta_1 = \beta_1 \sqrt{\left(\frac{\Delta \beta^*}{\beta^*}\right)^2 + \left(\frac{\Delta \beta_2}{1+\beta_2}\right)^2}$$

Derivation of Measurement Errors - CW Measurement:

ΔQ_2 − Uncertainty in Q_2 Determined from Decay Measurement

Parameter Uncertainties:

$$\Delta P_{im} = P_{im} \times \Delta P_{cal} + P_{\min}$$
$$\Delta P_{rm} = P_{rm} \times \Delta P_{cal} + P_{\min}$$
$$\Delta P_{tm} = P_{tm} \times \Delta P_{cal} + P_{\min}$$

$$\Delta P_{incident} = P_{incident}\sqrt{(\Delta C)^2 + \left(\frac{\Delta P_{im}}{P_{im}}\right)^2}$$

$$\Delta P_{reflected} = P_{reflected}\sqrt{(\Delta C)^2 + \left(\frac{\Delta P_{rm}}{P_{rm}}\right)^2}$$

$$\Delta P_{transmitted} = P_{transmitted}\sqrt{(\Delta C)^2 + \left(\frac{\Delta P_{tm}}{P_{tm}}\right)^2}$$

$$\Delta|\Gamma| = \frac{|\Gamma|}{2}\sqrt{\left(\frac{\Delta P_{reflected}}{P_{reflected}}\right)^2 + \left(\frac{\Delta P_{incident}}{P_{incident}}\right)^2}$$

$$\Delta Q_0 = Q_0\sqrt{\left(\frac{\Delta Q_2}{Q_2}\right)^2 + \left(\frac{\Delta P_{transmitted}}{P_{transmitted}}\right)^2 + \left(\frac{\Delta P_{loss}}{P_{loss}}\right)^2}$$

$$\Delta P_{loss} = \sqrt{(\Delta P_{incident})^2 + (\Delta P_{reflected})^2 + (\Delta P_{transmitted})^2}$$

$$\Delta \beta_2 = \beta_2\sqrt{\left(\frac{\Delta Q_0}{Q_0}\right)^2 + \left(\frac{\Delta Q_2}{Q_2}\right)^2}$$

$$\Delta \beta_1 = \beta_1\sqrt{\left(\frac{\Delta|\Gamma|}{1+|\Gamma|}\right)^2 + \left(\frac{\Delta|\Gamma|}{1-|\Gamma|}\right)^2}$$

$$\Delta Q_1 = Q_1\sqrt{\left(\frac{\Delta Q_0}{Q_0}\right)^2 + \left(\frac{\Delta \beta_1}{\beta_1}\right)^2}$$

$$\Delta Q_l = Q_l\sqrt{\left(\frac{\Delta Q_0}{Q_0}\right)^2 + \left(\frac{\Delta\beta_1{}^2 + \Delta\beta_2{}^2}{(1+\beta_1+\beta_2)^2}\right)}$$

$$\Delta E = \frac{E}{2}\sqrt{\left(\frac{\Delta Q_2}{Q_2}\right)^2 + \left(\frac{\Delta P_{transmitted}}{P_{transmitted}}\right)^2}$$